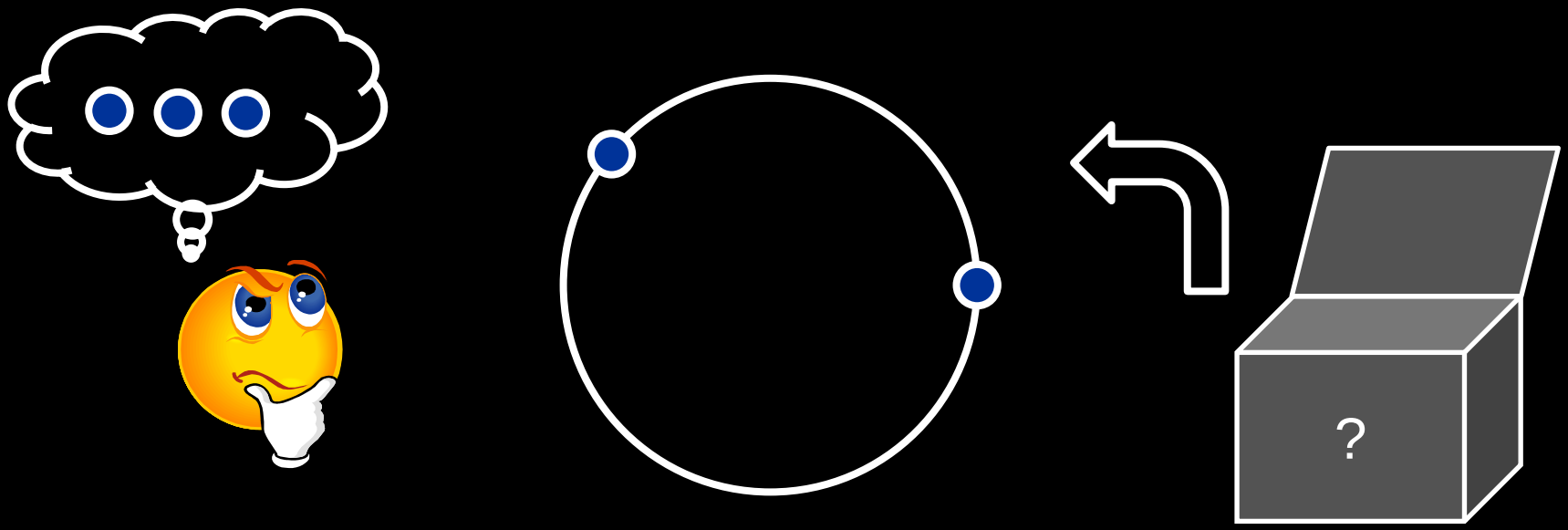
Online Weight Balancing on the Unit Circle



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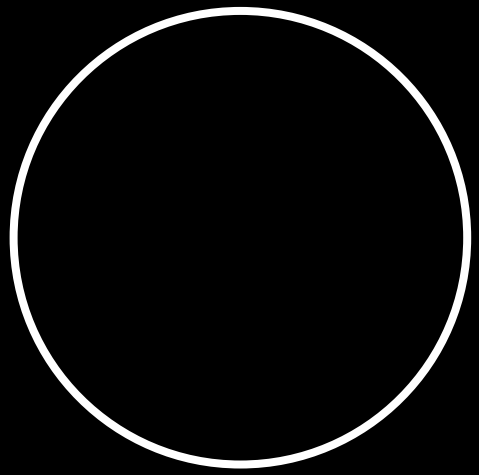
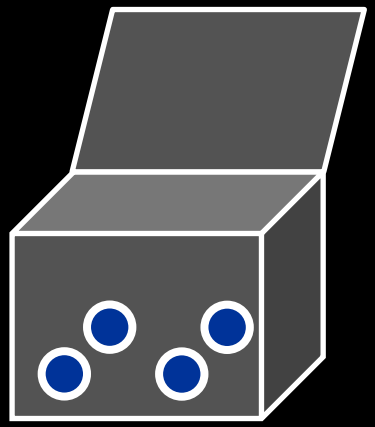
Toyohashi University of Technology

Online Weight Balancing

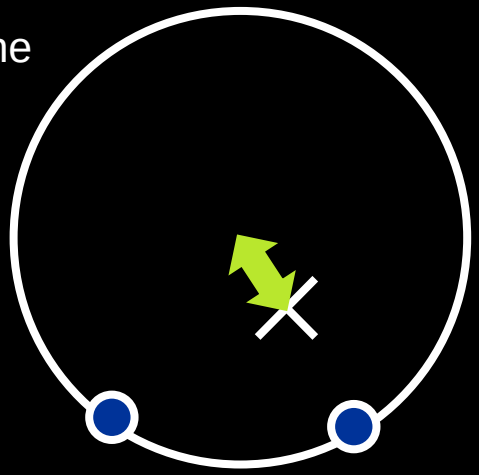


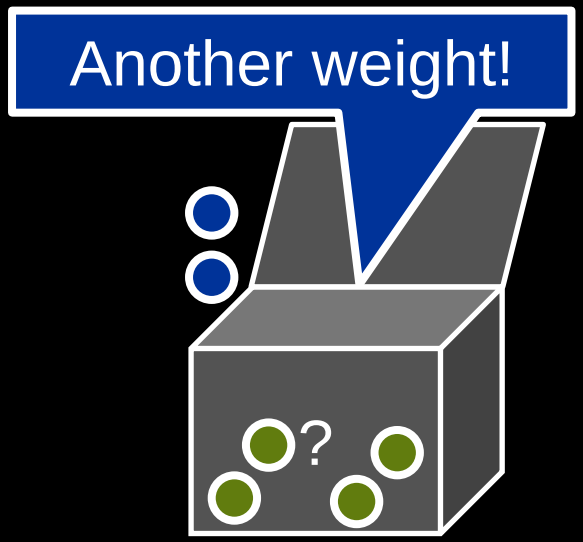


- Task: Place unit weights on unit circle
  - Number of weights unknown
  - Replacement forbidden
- Objective: Achive good balance

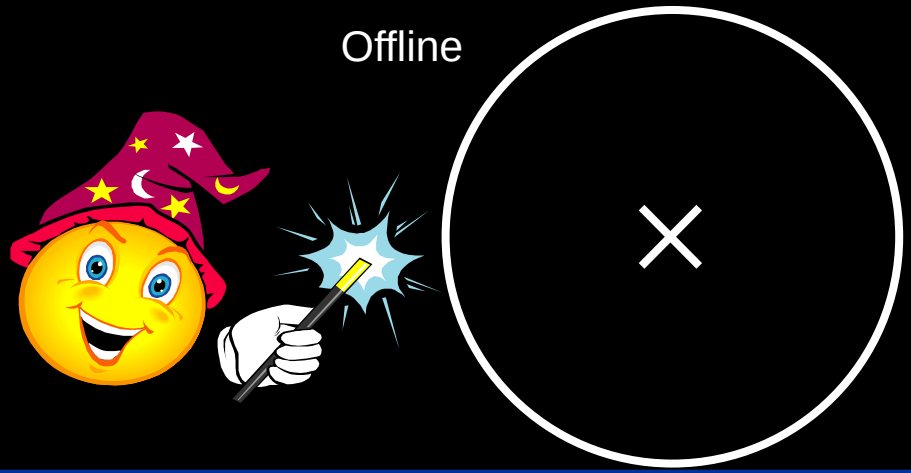
3 Weights in Box?

Online



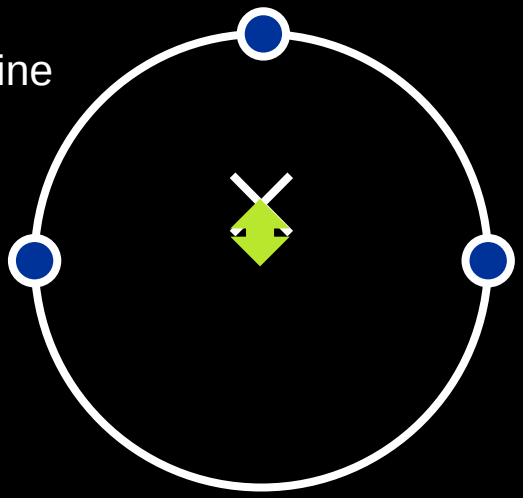


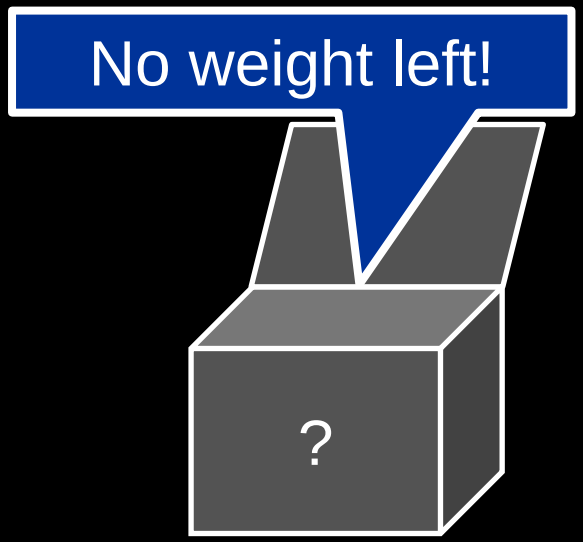
Offline



Then, 4 Weights in Box?

Online





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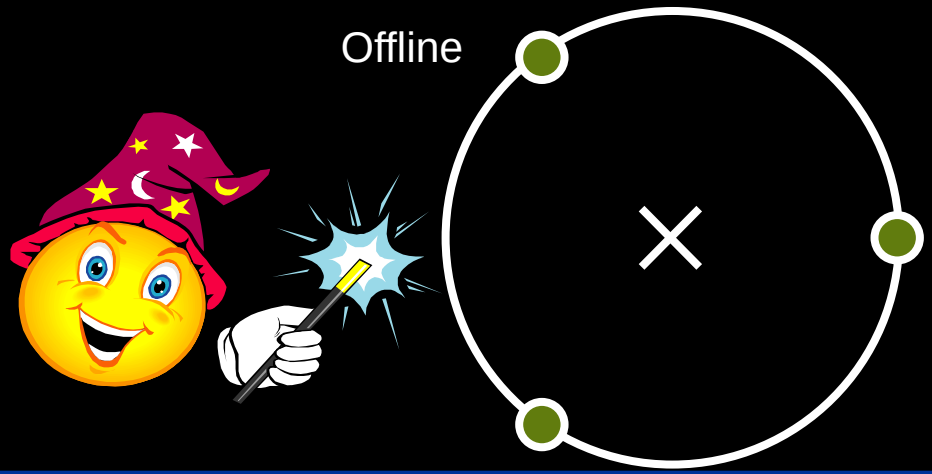
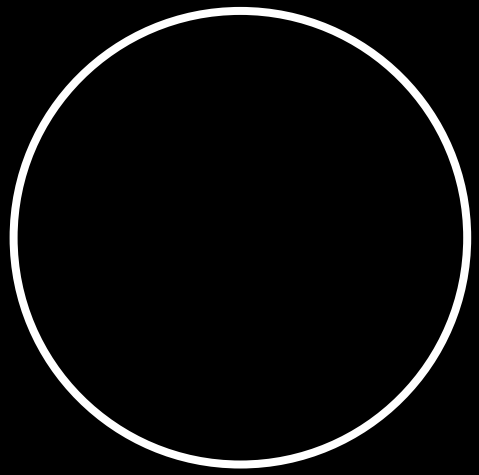
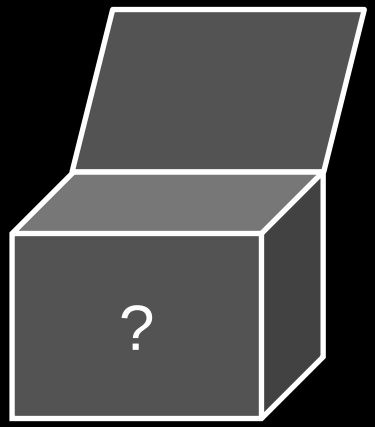


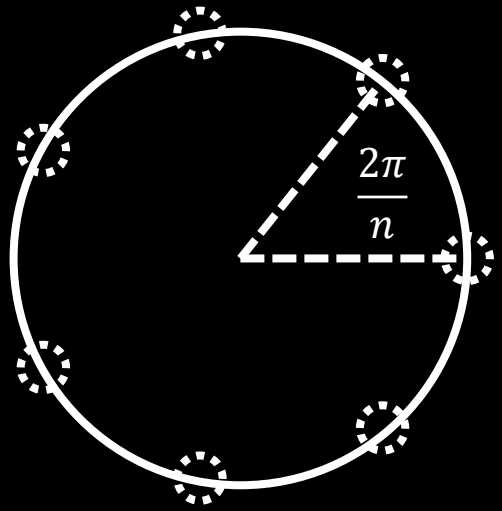
Online Weight Balancing: Basic Problem

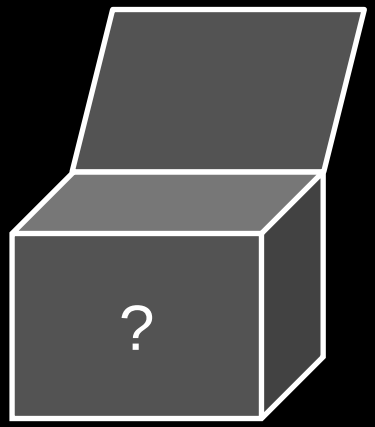




- Task: Place unit weights on unit circle
  - Number of weights unknown
  - Replacement forbidden
- Objective: Minimize distance(origin, center of mass)

Online Weight Balancing: n-Cyclotomic Problem





- Task: Place unit weights on unit circle
  - Number of weights unknown $\leq n$
  - Replacement forbidden
  - At most one weight at each candidate point
- Objective: Minimize distance(origin, center of mass)

Performance Measure of Strategy: Competitive Difference

Online

Offline

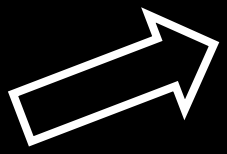
Definition:
Competitive difference = max(Onlinecost - Offlinecost)

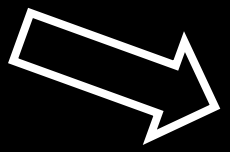
Why not competitive ratio?
Because Offlinecost = 0 often happens

# weights	1	2	…	…	…	…
Online	1	…	…	…	…	…
Offline	1	…	…	…	…	…
Difference	0	…	max	…	…	…

Motivation?

Task: Hang x pairs of socks





Application

Centrifuge machines

Picture at http://www.et.byu.edu/~wanderto/homealgaeproject/Harvesting%20Algae.html

Related Works

Our work is first to focus on online weight balancing. But in offline setting, for example:

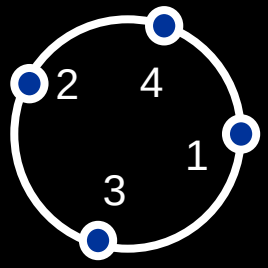
- Placement of rectangles to optimize position of center of mass [Kurebe&Miwa&Ibaraki 2007]
- Insertion of points into d-dimensional unit square to keep distance between any pair of points uniform [Teramoto&Asano&Katoh&Doerr 2007]

Our Results

place at $(cos\theta, sin\theta)$

Theorem 1 for basic problem:

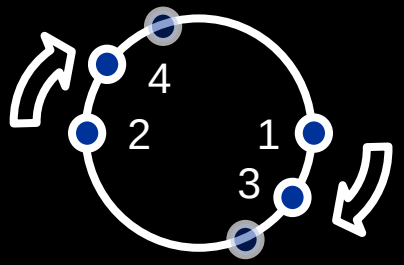
To place weights at $0, 2arccos\frac{1}{5}, \pi + arccos\frac{1}{5}, arccos\frac{1}{5}, \pi + arccos\frac{1}{5}, arccos\frac{1}{5}, \ldots$ is optimal. The competitive difference is $\frac{1}{5}$.



Theorem 2 for n-cyclotomic problem with n even:

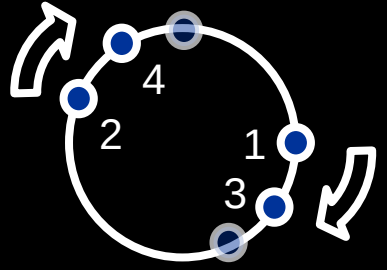
To place weights at $\frac{(j-1)\pi}{n}$ $(j = 1,3,5, \ldots, n-1)$ and $\frac{(j-1)\pi}{n} + \pi$ $(j = 2,4,6, \ldots, n)$ achieves competitive difference of $\frac{1}{3}$.

simple
but
not optimal



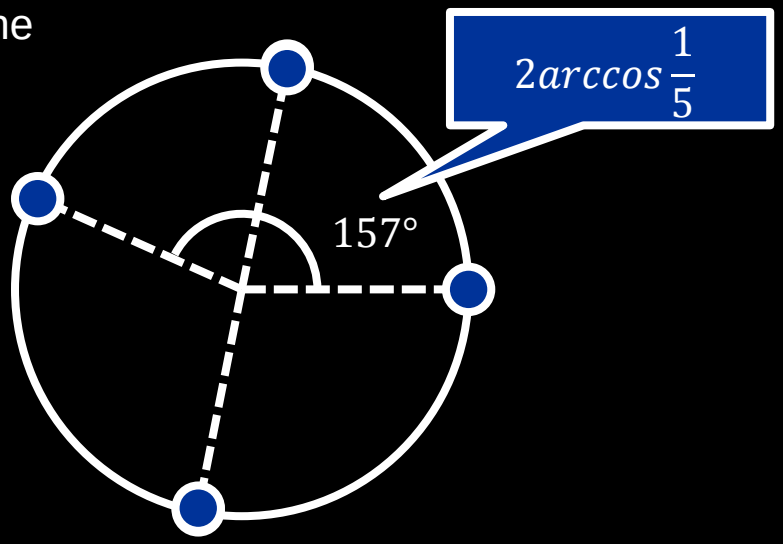
Theorem 3 for n-cyclotomic problem with n odd:

To place weights at $\frac{(j-1)(n-1)\pi}{n}$ $(j = 1,2,3, \ldots, n)$ achieves competitive difference of $\frac{1}{3cos\frac{\pi}{14}} \approx 0.34$.

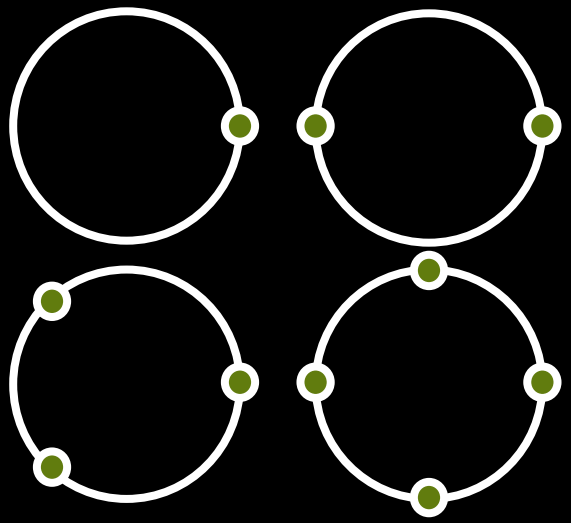


Optimal Strategy for Basic Problem

Online



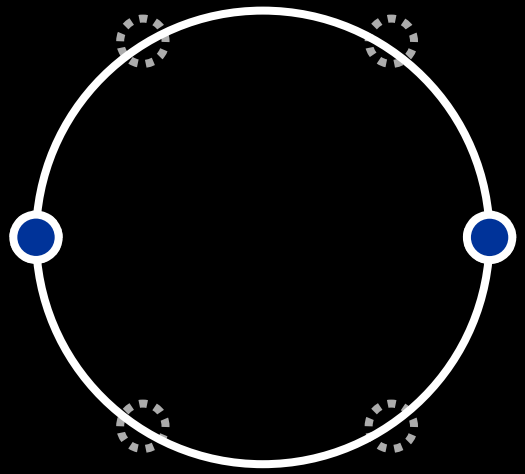
Offline



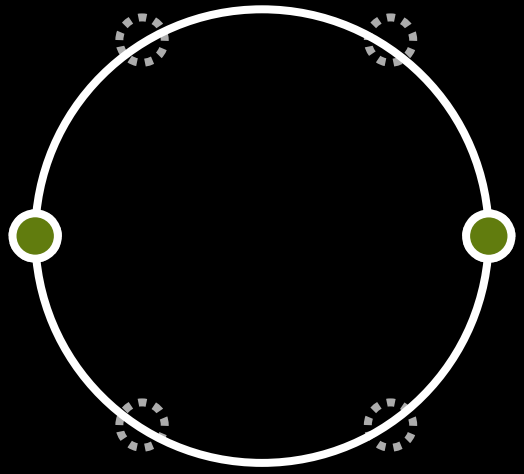
# weights	1	2	3	4	5	6	7	8
Online	1	0.2	0.2	0.1	0.12	0.07	0.09	0.05
Offline	1	0	0	0	0	0	0	0
Difference	0	0.2	0.2	0.1	0.12	0.07	0.09	0.05

Strategy for 6-Cyclotomic Problem

Online

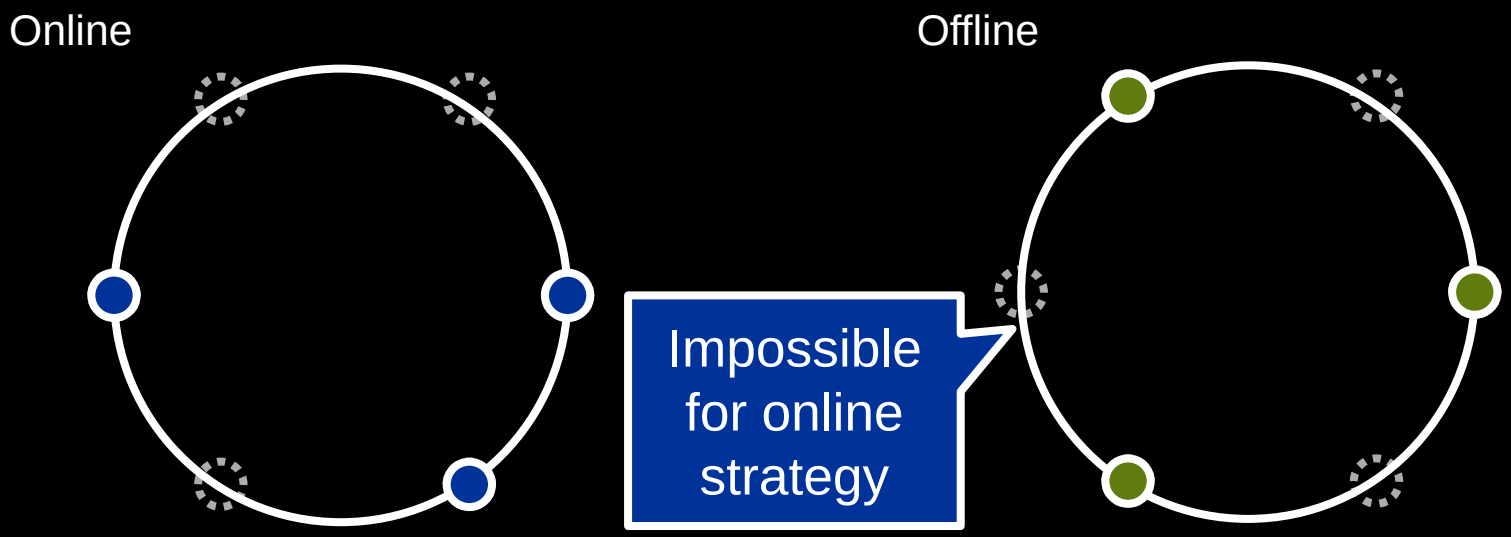


Offline



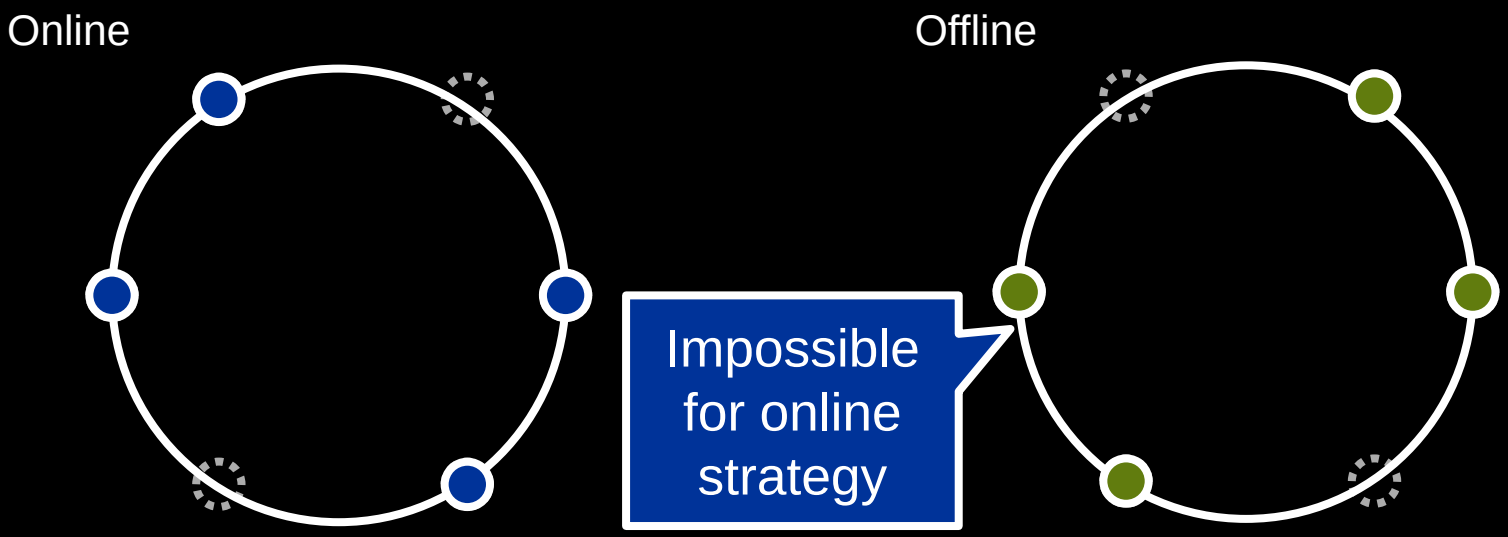
# weights	1	2				
Online	1	0				
Offline	1	0				
Difference	0	0				

Strategy for 6-Cyclotomic Problem



# weights	1	2	3			
Online	1	0	0.33			
Offline	1	0	0			
Difference	0	0	0.33			

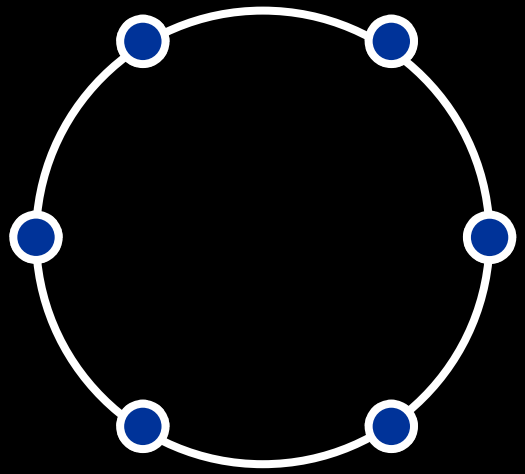
Strategy for 6-Cyclotomic Problem



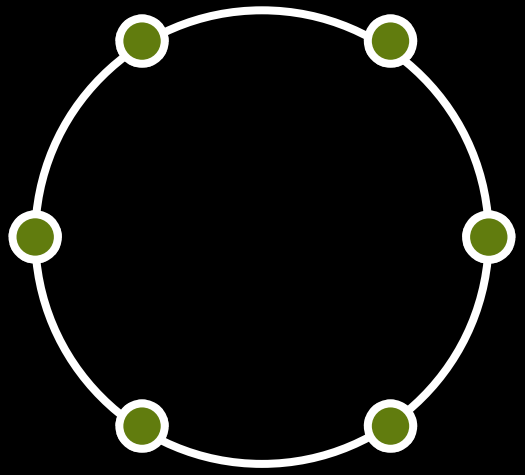
# weights	1	2	3	4		
Online	1	0	0.33	0		
Offline	1	0	0	0		
Difference	0	0	0.33	0		

Strategy for 6-Cyclotomic Problem

Online



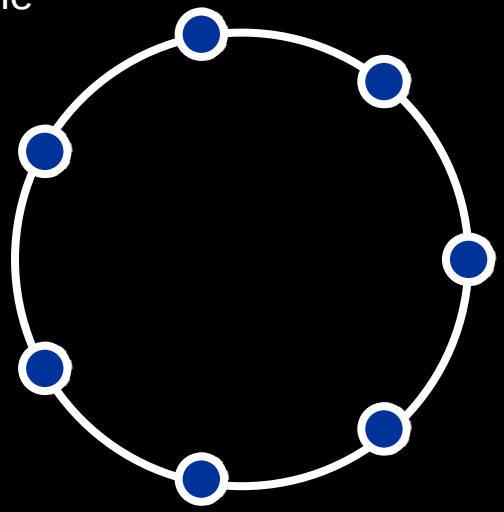
Offline



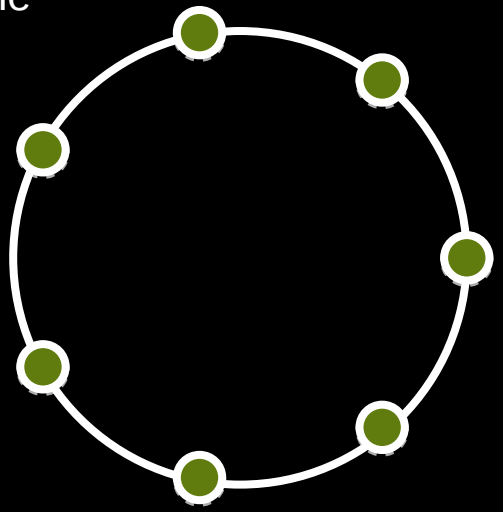
# weights	1	2	3	4	5	6
Online	1	0	0.33	0	0.2	0
Offline	1	0	0	0	0.2	0
Difference	0	0	0.33	0	0	0

Strategy for 7-Cyclotomic Problem

Online



Offline



# weights	1	2	3	4	5	6	7
Online	1	0.22	0.27	0.2	0.09	0.17	0
Offline	1	0.22	0.18	0.14	0.09	0.17	0
Difference	0	0	0.09	0.06	0	0	0

Summary

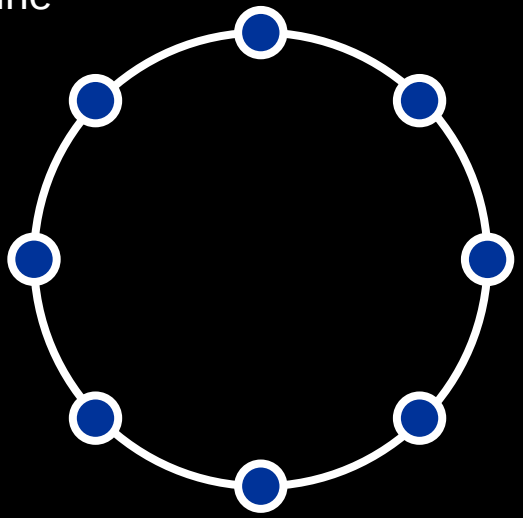
- For basic problem, we give optimal strategy
- For n-cyclotomic problem, we give simple strategy

Future Works

- For n-cyclotomic problem, better strategy?
  - For cases $2 \leq n \leq 8$, we have optimal strategy
  - For cases $n=6$ and 8, no strategy can achieve competitive difference of zero
- Another measure on balance?
- Arbitrary weights?

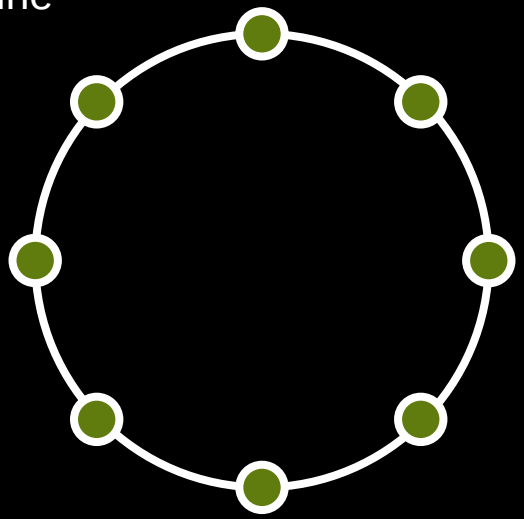
Strategy for 8-Cyclotomic Problem

Online



n = 8

Offline



# weights	1	2	3	4	5	6	7	8
Online	1	0	0.33	0	0.2	0	0.14	0
Offline	1	0	0.14	0	0.08	0	0.14	0
Difference	0	0	0.19	0	0.12	0	0	0