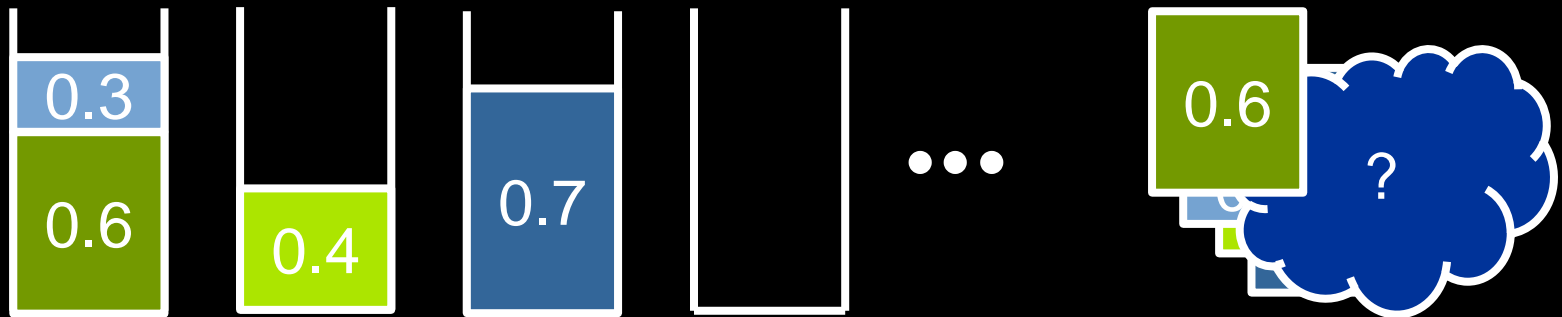


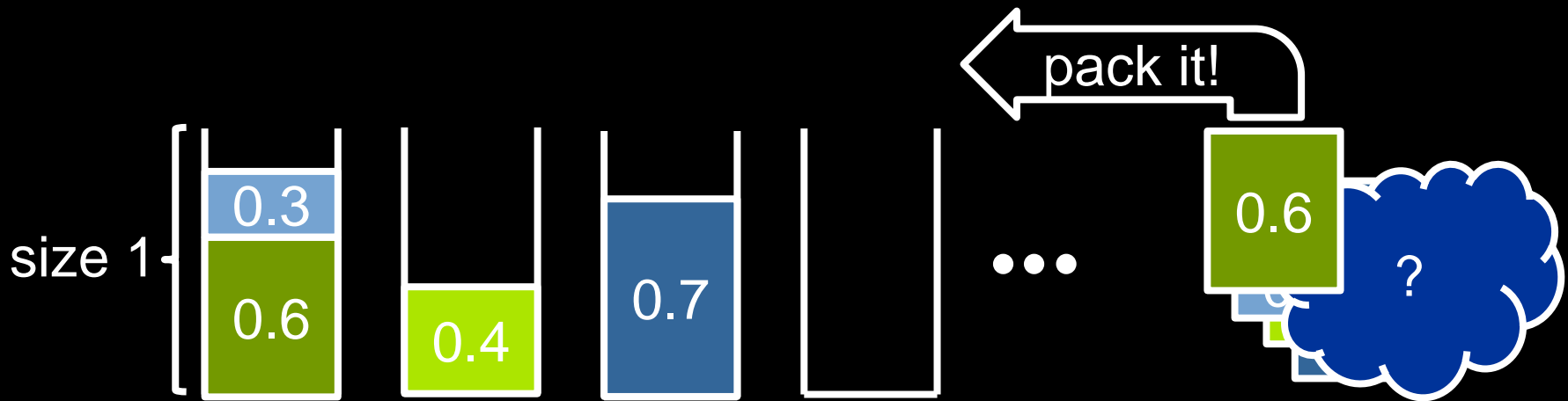
Improved Lower Bounds for the Online Bin Packing Problem with Cardinality Constraints



Hiroshi Fujiwara and Koji Kobayashi

Toyohashi University of Technology
National Institute of Informatics

Online Bin Packing Problem



- Pack **items** into **bins**
 - Items with size in $(0,1]$
 - Bins with size 1
- No information about future items
 - Size or the end of sequence
- Objective: **Minimize # bins used**

Example of Online Bin Packing Problem

Online algorithm:



OPT (optimal offline algorithm):



Example of Online Bin Packing Problem

Another online algorithm:

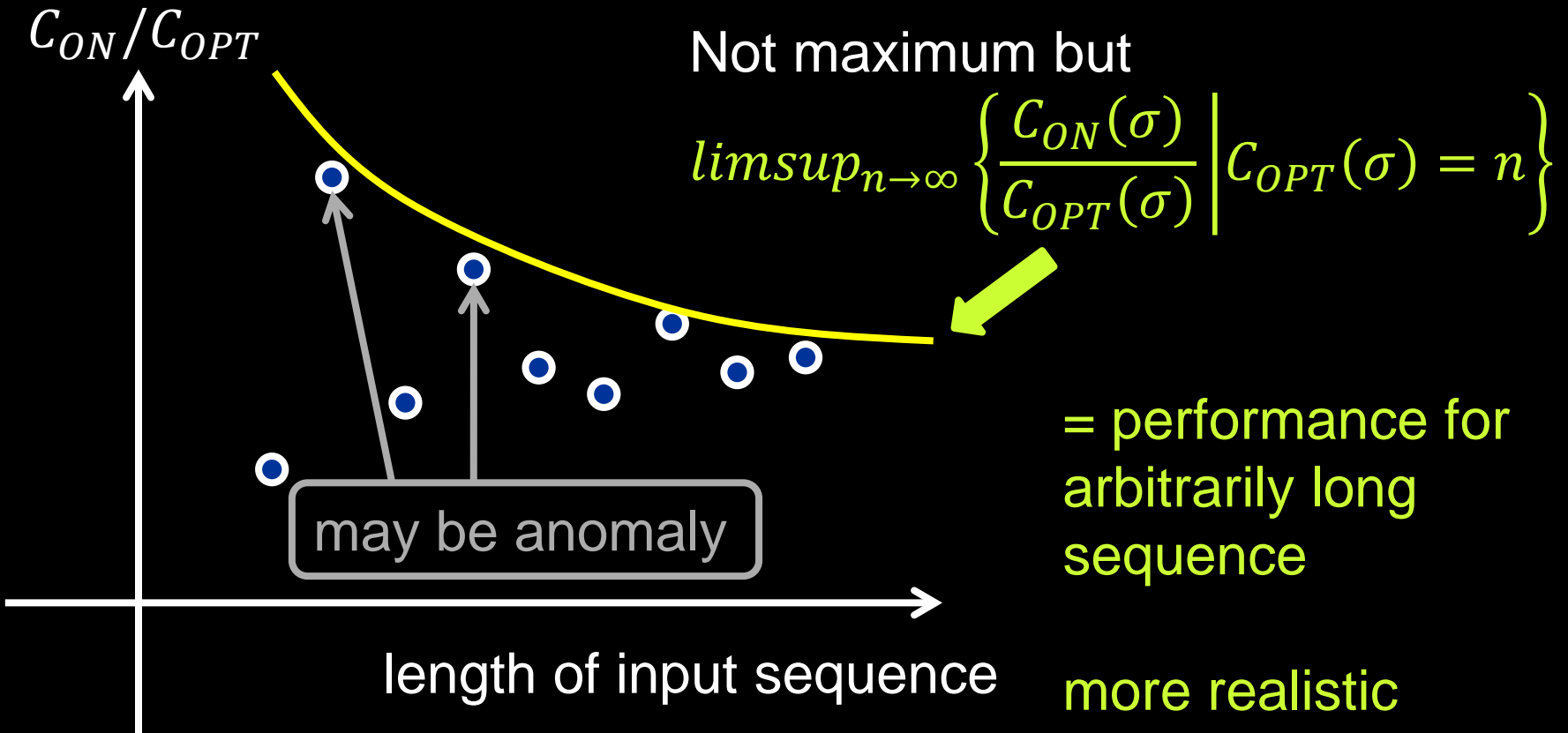


OPT:



We have used
2 times as many
bins!

Asymptotic Competitive Ratio



Online Bin Packing Problem with Cardinality Constraints

- Additional constraint:
Maximum # items in each bin
- For example, if cardinality constraint = 2,



Previous Results

cardinality constraint	lower bound	upper bound
2	1.41421 [Babel et al. 04]	1.44721 [Babel et al. 04]
3	1.5 [Babel et al. 04]	1.75 [Epstein 06]
4	1.33333 [van Vliet 92]	1.86843 [Epstein 06]
5	1.33333 [van Vliet 92]	1.93719 [Epstein 06]
6	1.5 [Yao 80]	1.99306 [Epstein 06]
7 to 9	1.5 [Yao 80]	2 [Babel et al. 04]
10 to 41	1.5 [Yao 80]	2 [Babel et al. 04]
≥ 42	1.539 [van Vliet 92]	2 [Babel et al. 04]
none	1.54037 [Balogh et al. 12]	1.58889 [Seiden 02]

Results explicitly considering
cardinality constraints

Our Results

cardinality constraint	lower bound	upper bound
2	1.41421 → 1.42764	1.44721 [Babel et al. 04]
3	1.5 [Babel et al. 04]	1.75 [Epstein 06]
4	1.33333 → 1.5	1.86843 [Epstein 06]
5	1.33333 → 1.47058	1.93719 [Epstein 06]
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10 to 41	1.5 → improved (see paper)	2 [Babel et al. 04]
≥ 42	1.539 [van Vliet 92]	2 [Babel et al. 04]
none	1.54037 [Balogh et al. 12]	1.58889 [Seiden 02]

input sequence with
3 scenarios

scheme for general
cardinality constraint
extending [van Vliet 92]

How to Get a Lower Bound

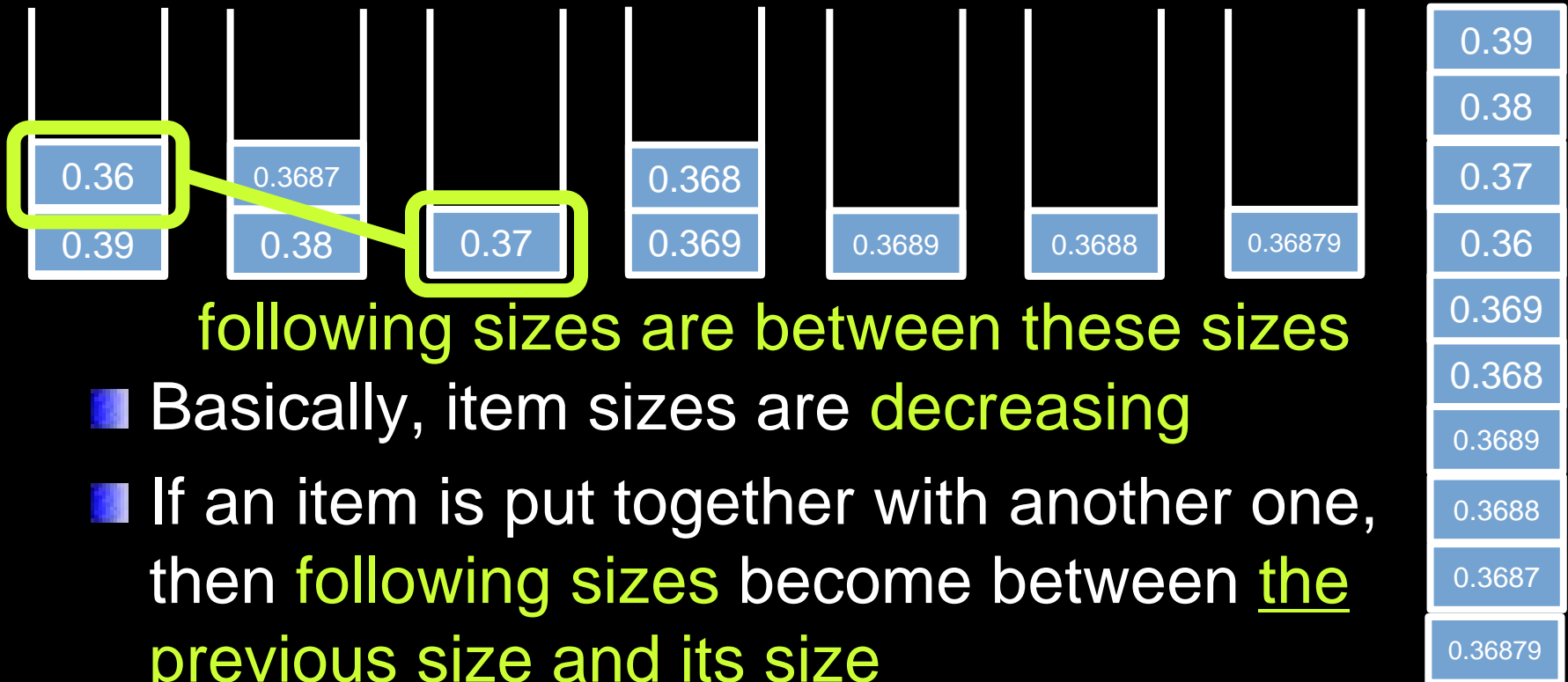
- Fix online algorithm arbitrarily
- Give input sequence that fools algorithm
- Note: its length should be taken arbitrarily; lower bound on **asymptotic** competitive ratio

- See lower bound of 1.41421 for cardinality constraint = 2 [Babel et al. 04]

Babel et al.'s Input Sequence for Cardinality Constraint = 2

[Babel&Chen&Kellerer&Kotov 04]

Arbitrary online algorithm:

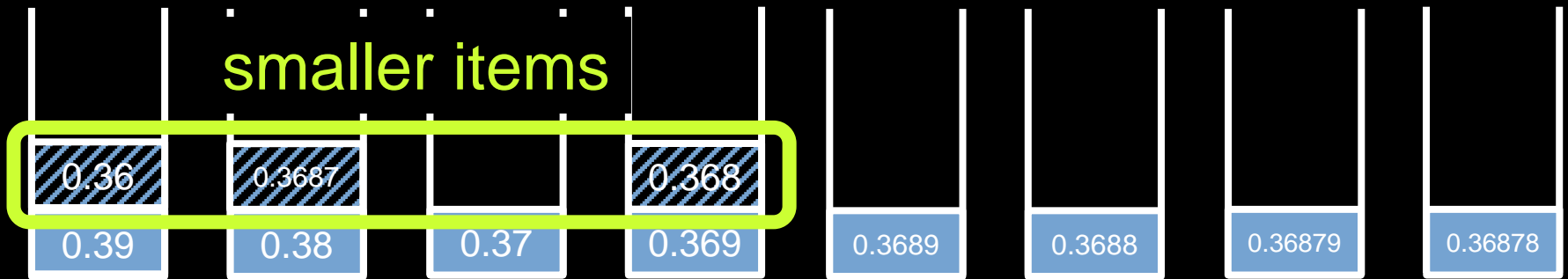


Babel et al.'s Input Sequence for Cardinality Constraint = 2

[Babel&Chen&Kellerer&Kotov 04]

Arbitrary online algorithm:

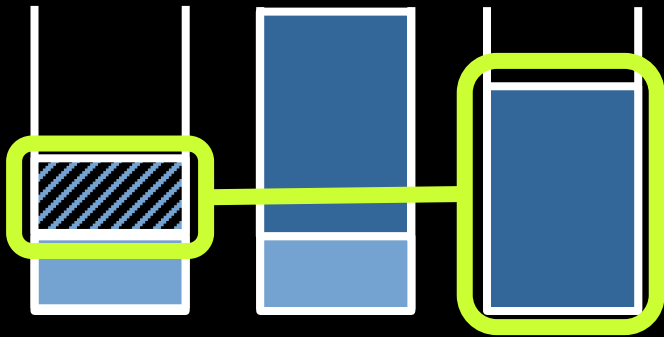
smaller items



Scenario 1

[Babel&Chen&Kellerer&Kotov 04]

Arbitrary online algorithm:



OPT:



fit how many?
as many as...

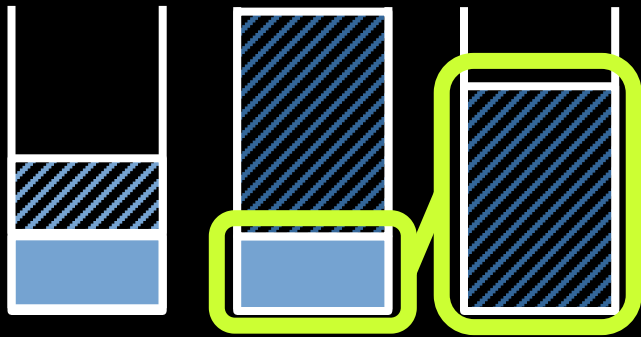
Note:

- From now and on, each figure represents set of such bins
- Imagine that there are many such bins!

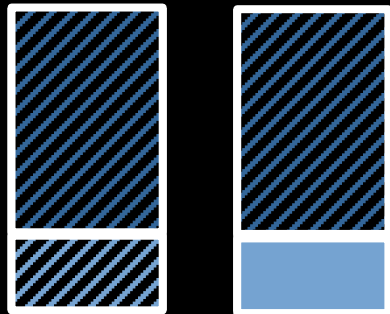
Scenario 1'

[Babel&Chen&Kellerer&Kotov 04]

Arbitrary online algorithm:



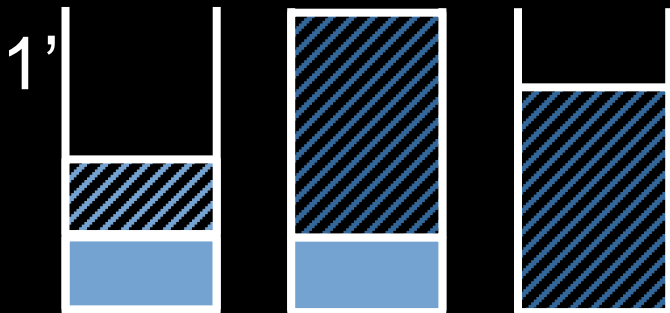
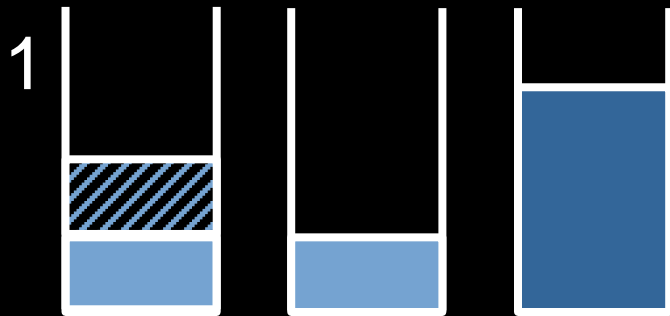
OPT:



fit how many?
as many as...

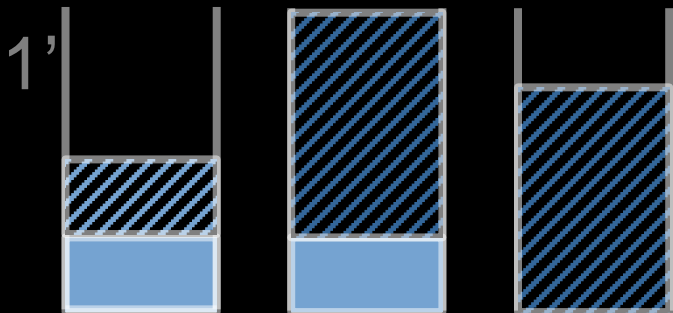
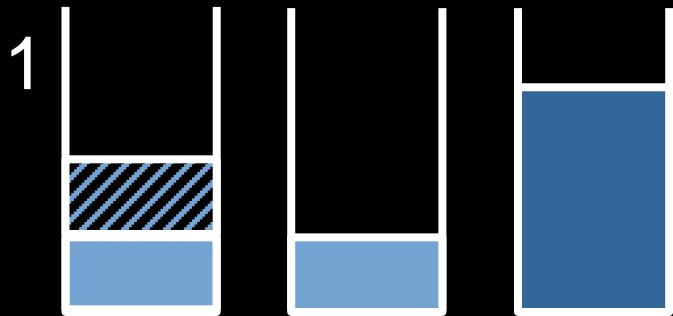
Babel et al.'s Lower Bound for Cardinality Constraint = 2

[Babel&Chen&Kellerer&Kotov 04]



- As shown, OPT makes only two-item bins
 - Fractions are negligible
- If algorithm makes more two-item bins, then ratio for $1'$ grows
- Either ratio exceeds 1.41421
 - Square root of 2

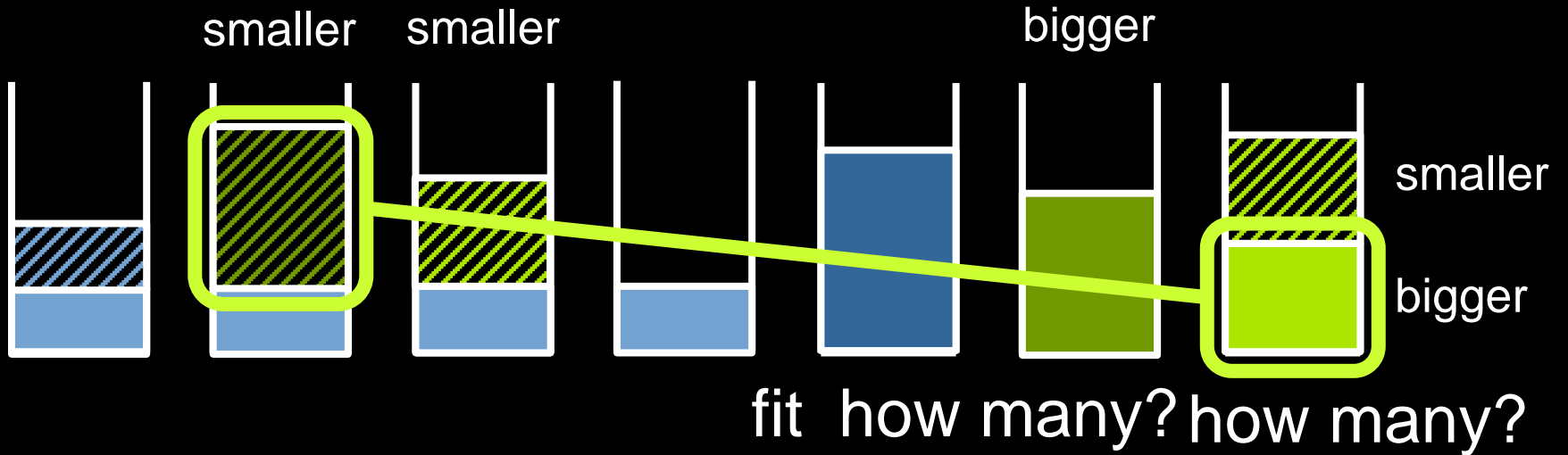
New Lower Bound for Cardinality Constraint = 2



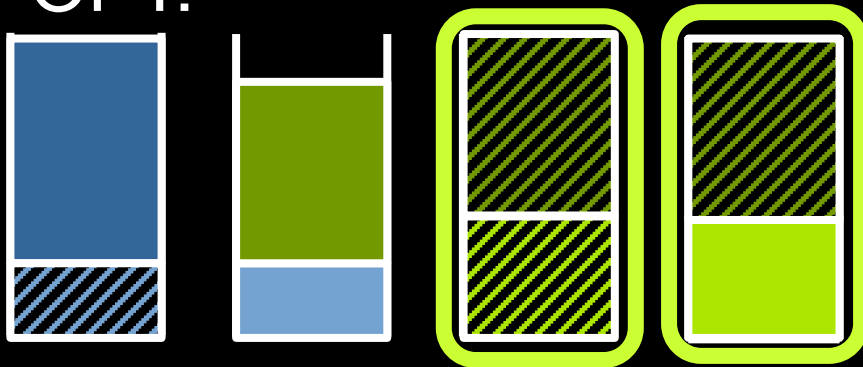
- Construct input sequence that fools algorithm more
- Using Scenario 1, design two more scenarios
- Algorithm cannot recognize which scenario happens



Scenario 2

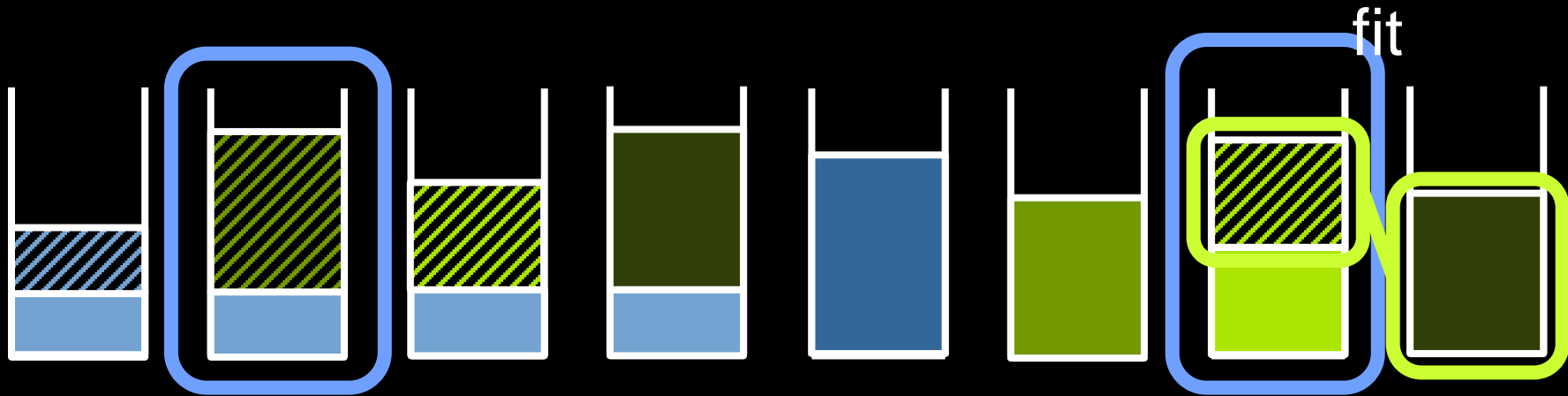


OPT:



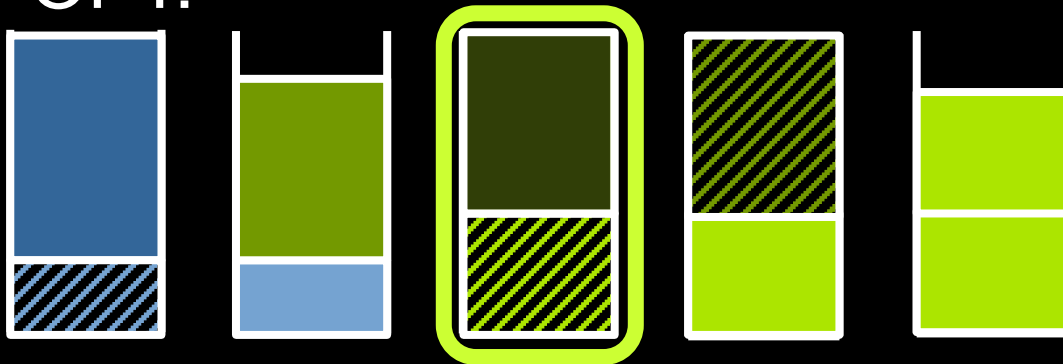
size > 1/2

Scenario 3

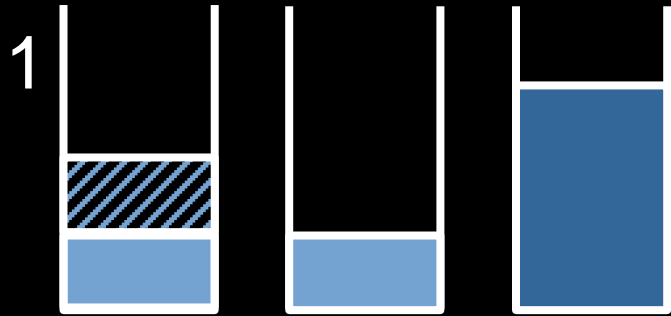


how many?
how many?

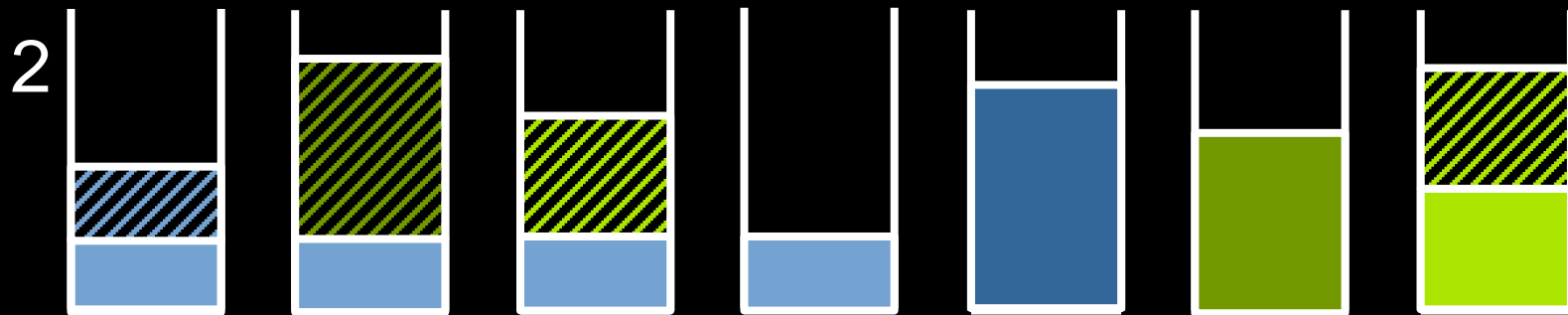
OPT:



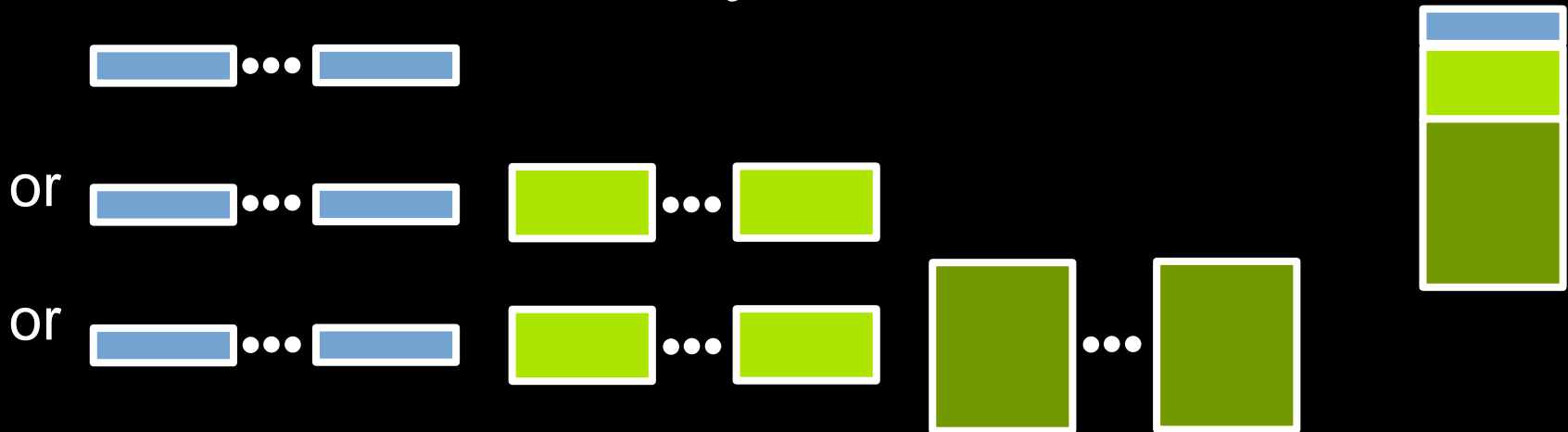
Finally



- As shown, OPT makes only two-item bins
- Either ratio exceeds 1.42764
 - Root of $2x^3 - 17x^2 + 30x - 14 = 0$



New Lower Bound for Cardinality Constraint ≥ 3



■ Formulate **linear program** [van Vliet 92]

■ Variable = # bins belonging to each type

■ Add cardinality constraint

■ Improve lower bounds for some cases of
cardinality constraints: 4, 5, 10 to 41, ...

Summary

cardinality constraint	lower bound	upper bound
2	1.41421 → 1.42764	1.44721 [Babel et al. 04]
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Future Works

- Better scheme for cardinality constraint ≥ 3
- Better upper bound?